M2 JUNE 13 INT

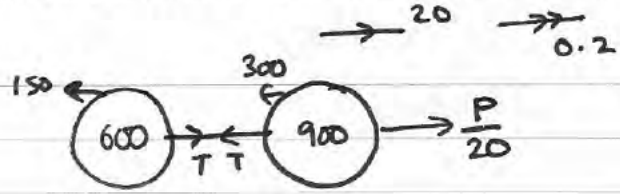
1. A caravan of mass 600 kg is towed by a car of mass 900 kg along a ... road. The towbar joining the car to the caravan is modelled as a light ro... road. The total resistance to motion of the car is modelled as having magn... N. The total resistance to motion of the caravan is modelled as having magnitude ... At a given instant the car and the caravan are moving with speed 20 m s^{-1} and acce... 0.2 m s^{-2}.

(a) Find the power being developed by the car's engine at this instant.

(5)

(b) Find the tension in the towbar at this instant.

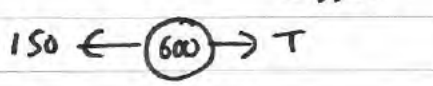
(2)



$\vec{Rf} = ma \Rightarrow \frac{P}{20} - 300 - 150 + T - T = 1500 \times 0.2$

$\therefore \frac{P}{20} = 750 \Rightarrow P = 15000 \text{ w}$

b) Caravan

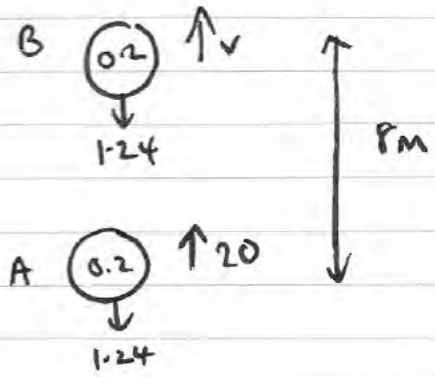


$\vec{Rf} = ma$

$T - 150 = 600 \times 0.2$

$\therefore T = 270 \text{N}$

2. A ball of mass 0.2 kg is projected vertically upwards from a point O ...ed 2(
The non-gravitational resistance acting on the ball is modelled as a ...
magnitude 1.24 N and the ball is modelled as a particle. Find, using th...
principle, the speed of the ball when it first reaches the point which is 8 ...ly
above O.



KE$_A$ − wd against n.g. Res = KE$_B$ + PE$_B$

$\frac{1}{2}m(20)^2 - \frac{1.24 \times 8}{(0.2)} = \frac{1}{2}mv^2 + mg \times 8$

$150.4 = \frac{1}{2}v^2 + 78.4 \quad \Rightarrow \frac{1}{2}v^2 = 72 \quad \therefore v = 12$

3. A particle P moves along a straight line in such a way that at time t s its velocity v m s^{-1} is given by

$$v = \frac{1}{2}t^2 - 3t + 4$$

Find

(a) the times when P is at rest,

(4)

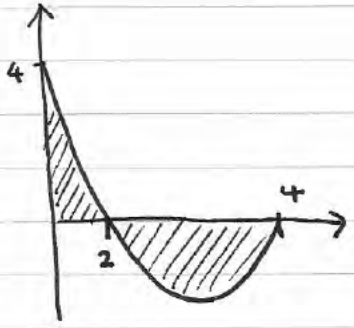
(b) the total distance travelled by P between $t = 0$ and $t = 4$.

(5)

a) $\frac{1}{2}t^2 - 3t + 4 = 0$ (×2) $t^2 - 6t + 8 = 0 \Rightarrow (t-4)(t-2) = 0$

$t = 4$ $t = 2$

b)



$s = \int v\,dt = \frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\ (+c)$

Area $= \left[\frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\right]_2^4 = \frac{8}{3} - \frac{-10}{3} = \frac{-2}{3}$

$\left[\frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\right]_0^2 = \frac{10}{3} - 0 = \frac{10}{3}$

$\therefore$ area $= \frac{10}{3} + \frac{2}{3} = 4 =$ total distance

4. A rough circular cylinder of radius $4a$ is fixed to a rough horizon[illegible] with [illegible]is horizontal. A uniform rod AB, of weight W and length $6a\sqrt{3}$, rests wit[illegible] in the plane and a point C of the rod against the cylinder. The vertical plan[illegible] is perpendicular to the axis of the cylinder. The rod is inclined at $60°$ to the [illegible], as shown in Figure 1.

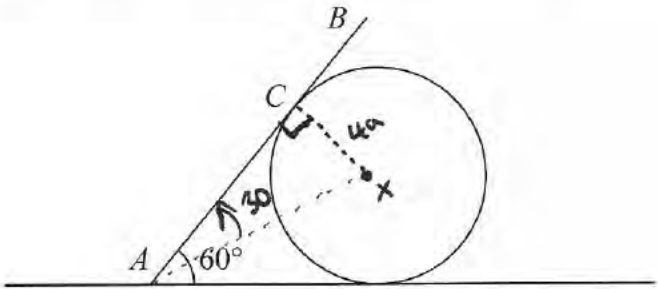


Figure 1

(a) Show that $AC = 4a\sqrt{3}$

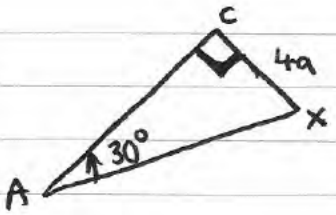
(2)

The coefficient of friction between the rod and the cylinder is $\frac{\sqrt{3}}{3}$ and the coefficient of friction between the rod and the plane is μ. Given that friction is limiting at both A and C,

(b) find the value of μ.

(9)

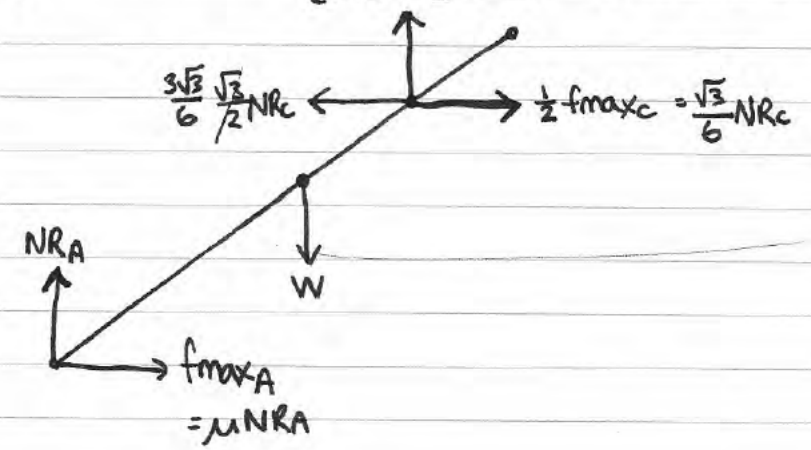
4a)

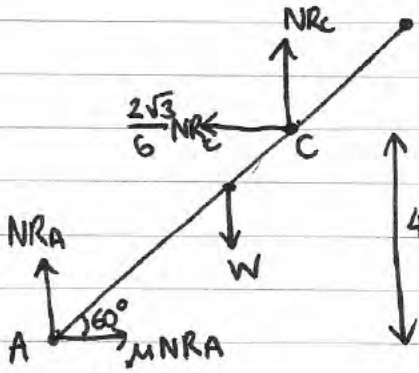


$\tan 30 = \frac{4a}{AC} \Rightarrow AC = \frac{4a}{\left(\frac{1}{\sqrt{3}}\right)} \Rightarrow$

b)

$\frac{1}{2}NR_C + \frac{\sqrt{3}}{2}f_{max_C} = \frac{1}{2}NR_C + \frac{1}{2}NR_C = NR_C$





$4\sqrt{3}a \sin 60 = \frac{4\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}a = 6a$

$\uparrow = \downarrow \Rightarrow NR_A + NR_C = W \quad (1)$

$\rightarrow = \leftarrow \Rightarrow \mu NR_A = \frac{2\sqrt{3}}{6}NR_C \quad (2)$

$3\sqrt{3}a \cos 60 = \frac{3}{2}\sqrt{3}a$

$4\sqrt{3}a \cos 60 = 2\sqrt{3}a$

A↻ $W \times \frac{3\sqrt{3}}{2}a = NR_C \times 2\sqrt{3}a + \frac{2\sqrt{3}}{6}NR_C \times 6a$

$W \times \frac{3\sqrt{3}}{2} = 4\sqrt{3} \times NR_C$

$\therefore W = \frac{8}{3}NR_C$ sub in (1)

$NR_A + NR_C = \frac{8}{3}NR_C \Rightarrow NR_A = \frac{5}{3}NR_C$ sub in (2)

$\mu \times \frac{5}{3}NR_C = \frac{2\sqrt{3}}{6}NR_C \Rightarrow \mu = \frac{6\sqrt{3}}{30} \therefore \mu = \frac{\sqrt{3}}{5}$

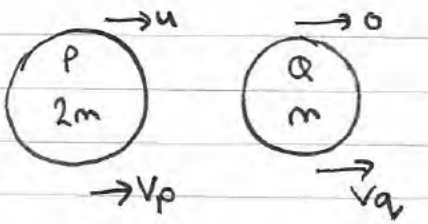
5. Two particles P and Q, of masses $2m$ and m respectively, are on a smo... zont... Particle Q is at rest and particle P collides directly with it when movi...

After the collision the total kinetic energy of the two particles is $\frac{3}{4}mu^2$. Find

(a) the speed of Q immediately after the collision,

(b) the coefficient of restitution between the particles.

(3)



KE before $= \frac{1}{2}(2m)u^2 = mu^2$

KE after $= \frac{3}{4}mu^2$

$\therefore \frac{1}{4}mu^2 =$ energy lost

$\frac{1}{2}(2m)V_p^2 + \frac{1}{2}mV_q^2 = \frac{3}{4}mu^2$

$\Rightarrow V_p^2 + \frac{1}{2}V_q^2 = \frac{3}{4}u^2 \Rightarrow 4V_p^2 + 2V_q^2 = 3u^2$ (1)

CLM $\Rightarrow 2mu = 2mV_p + mV_q \Rightarrow 2u = 2V_p + V_q$ (2)

$2V_p = 2u - V_q \Rightarrow 4V_p^2 = 4u^2 - 4uV_q + V_q^2$ sub in (1)

$(4u^2 - 4uV_q + V_q^2) + 2V_q^2 = 3u^2$

$u^2 - 4uV_q + 3V_q^2 = 0 \Rightarrow (u - 3V_q)(u - V_q) = 0$

$\therefore u = 3V_q \quad u = V_q$

$u \neq V_q \quad \therefore u = V_q$

b) $e = \frac{V_q - V_p}{u}$

(2) $- 2u = 2V_p + u$

$\Rightarrow u = 2V_p \Rightarrow V_p = \frac{1}{2}u$

$\Rightarrow e = \frac{u - \frac{1}{2}u}{u} = \frac{\frac{1}{2}u}{u} \quad \therefore e = \frac{1}{2}$

6.

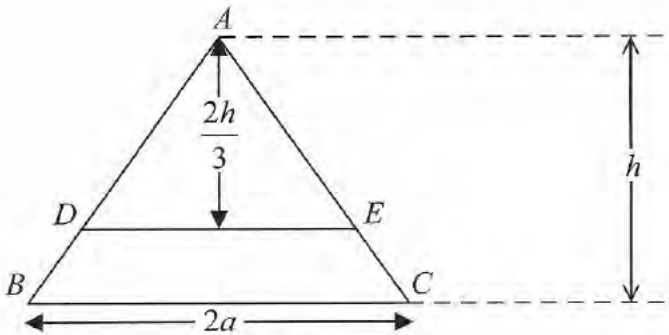


Figure 2

A uniform triangular lamina ABC of mass M is such that $AB = AC$, $BC = 2a$ and the distance of A from BC is h. A line, parallel to BC and at a distance $\frac{2h}{3}$ from A, cuts AB at D and cuts AC at E, as shown in Figure 2.

It is given that the mass of the trapezium $BCED$ is $\frac{5M}{9}$.

(a) Show that the centre of mass of the trapezium $BCED$ is $\frac{7h}{45}$ from BC.

(5)

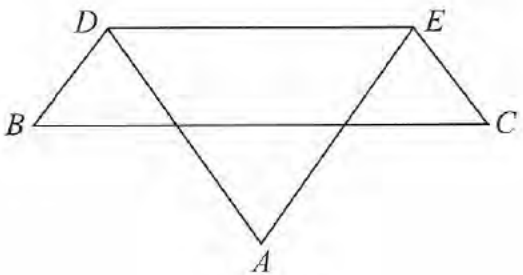


Figure 3

The portion ADE of the lamina is folded through 180° about DE to form the folded lamina shown in Figure 3.

(b) Find the distance of the centre of mass of the folded lamina from BC.

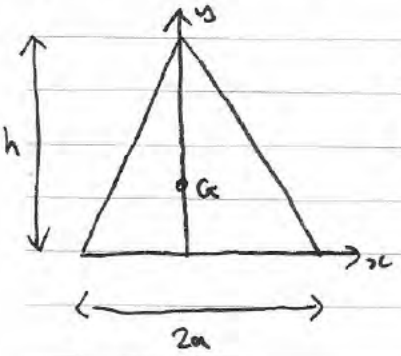
(4)

The folded lamina is freely suspended from D and hangs in equilibrium. The angle between DE and the downward vertical is α.

(c) Find $\tan \alpha$ in terms of a and h.

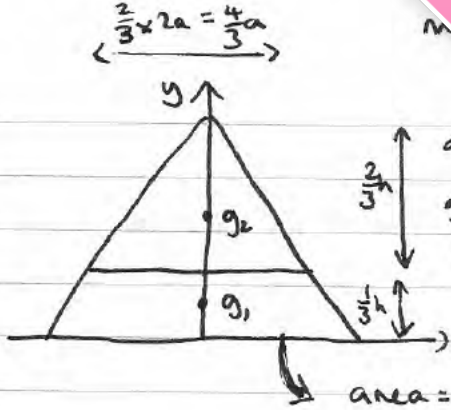
(4)

Question 6 continued



area = ah

$G(0, \frac{1}{3}h)$



area = $\frac{2}{3}a$

$g_2(0, \frac{1}{3}h + \frac{2}{9}h)$

$g_2(0, \frac{5}{9}h)$

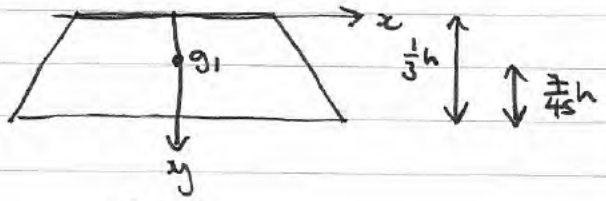
area = $\frac{5}{9}ah$

$g_1(0, \bar{y})$

$\curvearrowright x \quad \frac{5}{9}ah\,kg \times \bar{y} + \frac{4}{9}ah\,kg \times \frac{5}{9}h = ah\,kg \times \frac{1}{3}h$

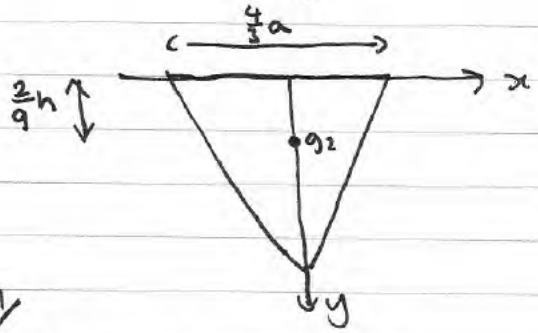
$\Rightarrow \frac{5}{9}5 + \frac{20}{81}h = \frac{27}{81}h \quad \Rightarrow \frac{45}{81}\bar{y} = \frac{7}{81}h \quad \therefore \bar{y} = \frac{7}{45}h$ #

b)

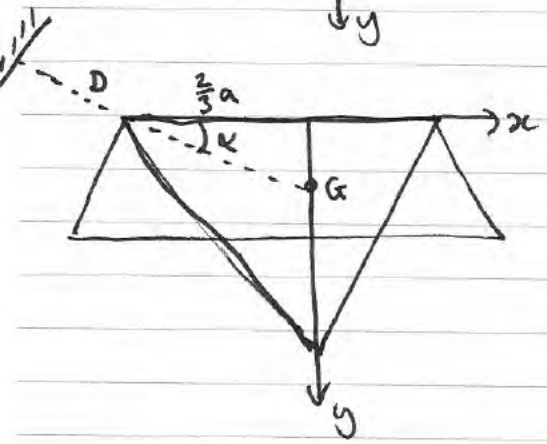


$\updownarrow = \frac{1}{3}h - \frac{7}{45}h = \frac{8}{45}h$

area = $\frac{5}{9}ah$ $g_1(0, \frac{8}{45}h)$



area = $\frac{4}{9}ah$ $g_2(0, \frac{2}{9}h)$



area = ah (combined!) $G(0, \bar{y})$

$\curvearrowright x \quad \frac{5}{9}ah\,kg \times \frac{8}{45}h + \frac{4}{9}ah\,kg \times \frac{2}{9}h = ah\,kg \times \bar{y}$

$\Rightarrow \frac{8}{81}h + \frac{8}{81}h = \bar{y} \quad \therefore \bar{y} = \frac{16}{81}h$

from BC = $\frac{1}{3}h - \frac{16}{81}h = \frac{11}{81}h$

c) $\tan\alpha = \frac{\frac{16}{81}h}{\frac{2}{3}a} = \frac{\frac{16}{81}h}{\frac{54}{81}a} = \frac{16}{54}\frac{h}{a} \quad \therefore \tan\alpha = \frac{8h}{27a}$

7.

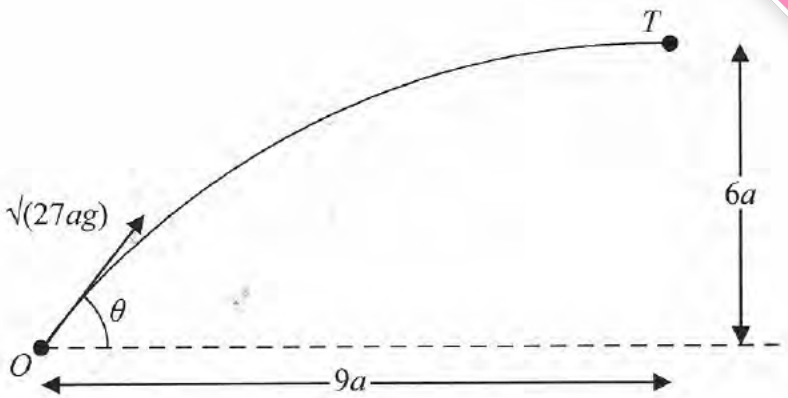


Figure 4

A small ball is projected from a fixed point O so as to hit a target T which is at a horizontal distance $9a$ from O and at a height $6a$ above the level of O. The ball is projected with speed $\sqrt{(27ag)}$ at an angle θ to the horizontal, as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

(a) Show that $\tan^2 \theta - 6 \tan \theta + 5 = 0$

(7)

The two possible angles of projection are θ_1 and θ_2, where $\theta_1 > \theta_2$.

(b) Find $\tan \theta_1$ and $\tan \theta_2$.

(3)

The particle is projected at the larger angle θ_1.

(c) Show that the time of flight from O to T is $\sqrt{\left(\frac{78a}{g}\right)}$.

(3)

(d) Find the speed of the particle immediately before it hits T.

(3)

$\vec{H}$ Vel $= \sqrt{(27ag)}\cos\theta$ dist $= 9a$ $\Rightarrow t = \dfrac{9a}{\sqrt{27ag}\cos\theta}$

$V\uparrow$ $s = 6a$, $u = \sqrt{27ag}\sin\theta$, v, $a = -g$, $t = \dfrac{9a}{\sqrt{27ag}\cos\theta}$

$s = ut + \frac{1}{2}at^2$

$$6a = \frac{9a\sqrt{27ag}\sin\theta}{\sqrt{27ag}\cos\theta} - \frac{1}{2}g\left(\frac{9a}{\sqrt{27ag}\cos\theta}\right)^2$$

$$6a = 9a\tan\theta - \frac{1}{2}g\left(\frac{81a^2}{27ag\cos^2\theta}\right)$$

$$\Rightarrow 6 = 9\tan\theta - \frac{3}{2}\sec^2\theta$$

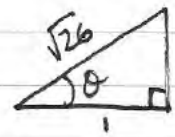
$$\Rightarrow 6 = 9\tan\theta - \frac{3}{2}(\tan^2\theta + 1)$$

$$\Rightarrow \frac{3}{2}\tan^2\theta - 9\tan\theta + \frac{15}{2} = 0$$

$$\therefore \tan^2\theta - 6\tan\theta + 5 = 0 \ \#$$

b) $(\tan\theta - 5)(\tan\theta - 1) = 0 \Rightarrow \tan\theta_1 = 5 \quad \tan\theta_2 = 1 \quad \theta_1 = 78.7 \quad \theta_2 = 45$

c) $\tan\theta = \frac{5}{1}$



$\cos\theta = \frac{1}{\sqrt{26}}$

$$\Rightarrow t = \frac{9a}{\sqrt{\frac{27ag}{26}}} = \sqrt{\frac{81a^2 \times 26}{27ag}} = \sqrt{\frac{78a}{g}}$$

d) $V_v\uparrow$ $\quad v^2 = u^2 + 2as \Rightarrow V\uparrow^2 = 27ag\sin^2\theta - 2g \times 6a$

$$\Rightarrow V^2\uparrow = 27ag\sin^2\theta - 12ga$$

$\text{Speed}^2 = V_v{}^2\uparrow + \vec{V_h}^2$

$\text{speed}^2 = 27ag\sin^2\theta - 12ga + 27ag\cos^2\theta = 27ag - 12ga$

$\therefore \text{Speed}^2 = 15ag \quad \therefore \text{Speed} = \sqrt{15ag}$